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A NEW METHOD FOR SOLVING DIFFRACTION PROBLEMS

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/Digest. Figures referred to are appended./

The essence of the proposed method follows: the given diffraction problem is formulated as a boundary-value problem with definite conditions on the boundaries of a certain region. This region is transformed (real) so that the boundary conditions are preserved, but so that the field in the transformed region can immediately be expressed in quadratures as in the elementary problem on radiation. With the aid of formulas, expressing the chosen transformation of the region, which result from the solution of the auxiliary problem of radiation, one obtains the desired solution of the diffraction problem.

The following example will clarify the method: Let us seek the field that arises when a single two-dimensional discrete pressure wave is diffracted by a rectilinear edge of a semi-infinite screen. The wave is propagated parallel to the plane of the screen and its front reaches the screen at moment $t=0$. Beyond the front of the single wave the pressure equals zero and behind the front it is unity. The diffraction phenomenon develops in a cylinder of radius ct. The axis of the cylinder is the edge of the screen; the problem is two-dimensional. The boundary conditions are these: on the upper semicircle we have $p=1$ (Figure 1) and on the lower semicircle $p=0$. At both places of the discontinuity we have AC, $\partial p/\partial n=0$ (i. e., the screen is assumed to be rigid).

For these boundary conditions the pressure must satisfy the wave equation:

$$\partial^2 p/\partial r^2+\frac{1}{r}\,\partial p/\partial r+\frac{1}{r^2}\,\partial^2 p/\partial a^2-\frac{1}{c^2}\,\partial^2 p/\partial t^2=0 \qquad (1)$$

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Let us consider the auxiliary problem: Let the semiplane be covered at t=0 by sources of an intensity such that if the entire plane were covered by these sources then it would radiate a single pressure wave. In other words, the normal velocity must be chosen in the form:

$$v = w^{-1}\cdot\sigma_0(t) = 0 \quad (t<0)$$
$$= w^{-1} \quad (t \geq 0), \qquad (2)$$

where $w=\rho_0 c$ is the wave resistance of the medium. The phenomenon is pictured in Figure 2. The edge of the radiating semiplane is located at A'. The semiplane left of A' is rigid and immobile. Boundary conditions are: p=1 on the arc C'B'; p=0 on arc B'D'; $\partial p/\partial n=0$ on D'A' and A'C'. The field within the semicircle C'B'D'A'C' is easily found.

We find from Figure 3:

$$dp = \rho_0 \dot{v}(t - R/c)\, dS/2\pi R \quad \& \quad dS = 2\theta a\, da = 2\theta R\, dR.$$

Furthermore,

$$\theta = \arccos \frac{\cos\beta}{\sqrt{(R/\rho)^2 - \sin^2\beta}}$$

and we have, taking (2) into mind:

$$p = \frac{1}{\pi}\int_{\rho/c}^{\infty} \theta(\tau)\, s_1(t-\tau)\, d\tau = \frac{1}{\pi}\,\theta(t)\cdot\sigma_0\,(t - \rho/c);$$

thus, within the semicircle $\rho \leq ct$ the pressure is expressed by:

$$p = \frac{1}{\pi}\arccos \frac{\pm\cos\beta}{\sqrt{(ct/\rho)^2 - \sin^2\beta}}, \qquad (3)$$

which represents the solution of the auxiliary problem of radiation.

Comparing the boundary conditions of the main problem and the auxiliary problem, we see that the boundary conditions of both problems coincide if we double the angles of the second problem. Here Figure 2 goes over to Figure 1. Consequently, one of the formulas giving the transformation of a semicircle into a circle has the form:

$$\alpha = 2\beta \qquad (4)$$

Let us find the transformation formulas of the radii. By the substitution cosh z=ct/r we convert the wave equation (1) into Laplace's equation $\partial^2\bar{p}/\partial z^2 + \partial^2\bar{p}/\partial\alpha^2 = 0$. Obviously, we can multiply both arguments by one and the same number without changing this equation. Introducing instead of α a new angle 2β we must correspondingly replace z by z=2y where cosh y = ct/ρ so that cosh z = cosh 2y = 2 cosh²y − 1 and we obtain the transformation formula of radii

$$ct/r = 2(ct/\rho)^2 - 1. \qquad (5)$$

Formulas (4) and (5) gives the transformation of the semicircle into a circle which is invariant relative to the wave equation.

It remains to substitute (4) and (5) into (3) in order to obtain the solution of our diffraction problem. We find:

$$p = \frac{1}{\pi}\arccos \frac{\pm\sqrt{2}\cos\alpha/2}{\sqrt{\frac{ct}{r} + \cos\alpha}} \qquad (6)$$

Formula 6 was obtained earlier by another method of the author (see Zhur Tekh Fiz, No 19, 828, 1949).

[Appended figures follow.]

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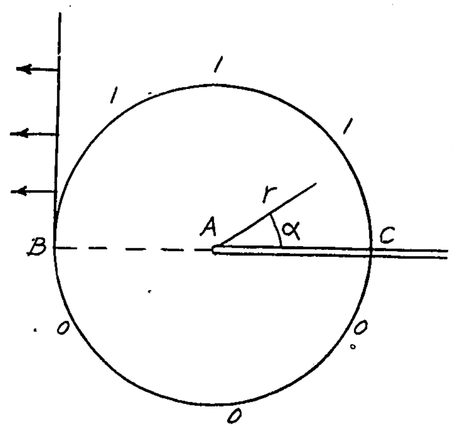

Figure 1

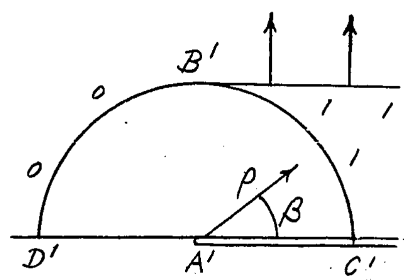

Figure 2

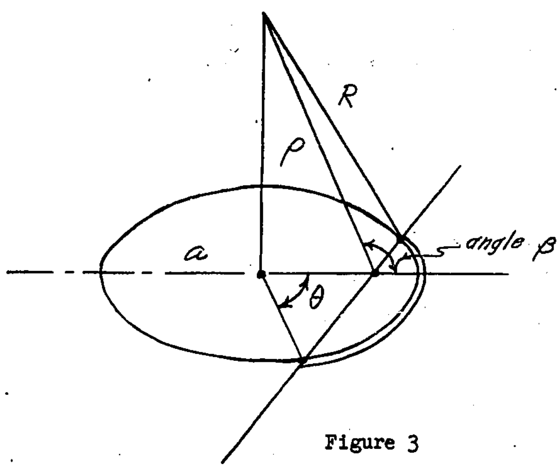

Figure 3

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